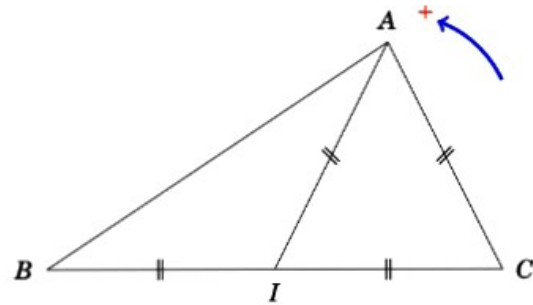


تمرين 1

أعطي القيس الرئيسي للزوايا التالية (حسب الشكل) :

$$(\overrightarrow{AC}, \overrightarrow{AB}) \quad (\overrightarrow{BI}, \overrightarrow{BA}) \quad (\overrightarrow{IB}, \overrightarrow{IA}) \quad (\overrightarrow{CA}, \overrightarrow{CI})$$



تمرين 2

بين أن x و y هما قيسين لنفس الزاوية الموجهة :

$$x = 123\pi, y = ; x = \frac{115\pi}{2}, y = \frac{729\pi}{6} ; x = \frac{11\pi}{4}, y = \frac{-5\pi}{4}$$

$$x = \frac{-3\pi}{2}, y = \frac{\pi}{2} ; 3\pi$$

تمرين 3

عين في كل حالة من الحالات التالية القيس الرئيسي للزاوية التي قياسها

$$\alpha = \frac{14\pi}{3} ; \alpha = \frac{35\pi}{2} ; \alpha = \frac{721\pi}{5} ; \alpha = \frac{2012\pi}{3}$$

على الدائرة المثلثية :

تمرين 4

في المستوي الموجه لدينا : $(\overrightarrow{U}; \overrightarrow{V}) = \frac{\pi}{3}$

عين قياسا لكل زاوية من الزوايا الموجهة التالية : $(3\overrightarrow{V}; 2\overrightarrow{U})$ $(\overrightarrow{V}; \overrightarrow{U})$ $(-\overrightarrow{V}; -\overrightarrow{U})$ $(-3\overrightarrow{V}; 7\overrightarrow{U})$ $(2\overrightarrow{V}; -3\overrightarrow{U})$

تمرين 5

لتكن (C) الدائرة المثلثية مرفقة بمعلم متعامد ومتجانس $(O, \vec{i}, \vec{j})$

لتكن A و B نقطتين من (C) حيث : $(\overrightarrow{OI}, \overrightarrow{OA}) = \frac{\pi}{6}$

$$(\overrightarrow{OI}, \overrightarrow{OB}) = \frac{3\pi}{4}$$

عين قياسا للزوايا الموجهة : $(\overrightarrow{OJ}, \overrightarrow{OB})$ $(\overrightarrow{OJ}, \overrightarrow{OA})$ $(\overrightarrow{OA}, \overrightarrow{OB})$

تمرين 6

DCB مثلث قائم في D و لتكن A نقطة من قطعة المستقيم [DB] ،

حيث : $AC = AB = 2\text{cm}$

ولدينا : $(\overrightarrow{AB}, \overrightarrow{AC}) = \frac{5\pi}{6} + 2k\pi \quad k \in \mathbb{Z}$

(1) أثبت الشكلى .

(2) أحسب كل من $(\overrightarrow{AC}, \overrightarrow{CB})$; $(\overrightarrow{CA}, \overrightarrow{CB})$; $(\overrightarrow{BC}, \overrightarrow{BA})$

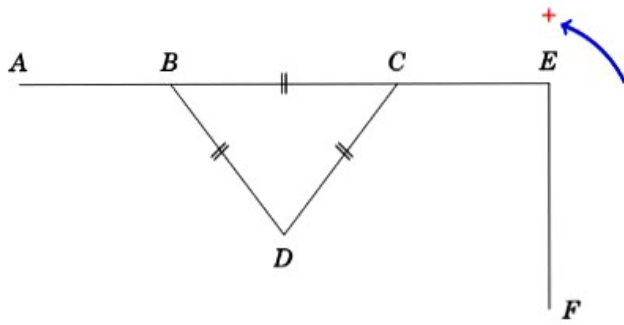
(3) أحسب BC ، DA ، DB ، DC ، ثم إستنتج $\sin \frac{\pi}{12}$ و

$$\cos \frac{\pi}{12}$$

تمرين 7

عين قياسا للزوايا الموجهة التالية :

$$(\overrightarrow{AB}, \overrightarrow{BD}) ; (\overrightarrow{CE}, \overrightarrow{CD}) ; (\overrightarrow{CE}, \overrightarrow{EF}) ; (\overrightarrow{BD}, \overrightarrow{EF})$$



تمرين 8

لدينا : $\cos x = \frac{1}{2}$

$$(1) \text{ أحسب } \sin(\pi - x) ; \cos\left(\frac{\pi}{2} - x\right) ; \sin\left(\frac{\pi}{2} - x\right) ; \sin x$$

$$(2) \text{ أحسب } \tan(\pi - x) ; \tan\left(\frac{\pi}{2} - x\right) ; \tan x$$

تمرين 9

أحسب A ، B و C حيث :

$$A = \cos\left(\frac{\pi}{2} + x\right) + \sin(\pi - x) + \cos\left(\frac{\pi}{2} - x\right) - \sin(-x)$$

$$B = \cos(\pi + x) + \cos\left(\frac{3\pi}{2} + x\right) + \sin(3\pi - x) + \sin\left(\frac{5\pi}{2} - x\right)$$

$$C = \cos\left(\frac{7\pi}{2} + x\right) + \cos\left(\frac{7\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} + x\right) - \sin\left(\frac{\pi}{2} - x\right)$$

تمرين 10

حل في المجال $[0; 2\pi]$ المعادلات التالية :

$$\cos -2x = -\frac{\sqrt{3}}{2} , \cos 3x = -\frac{\sqrt{2}}{2} , \cos x = \frac{\sqrt{3}}{2}$$

$$\sin -2x = -\frac{\sqrt{3}}{2} , \sin 3x = -\frac{\sqrt{2}}{2} , \sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos x + \sin x = 0 , \cos\left(x - \frac{\pi}{4}\right) = \sin\left(3x + \frac{\pi}{2}\right)$$

$$\sin^2 x - \sin x - 6 = 0 ,$$

تمرين 11

حل في $\mathbb{R}$ المعادلات التالية :

$$\sin 3x = \cos\left(x - \frac{\pi}{4}\right) , \sin 2x = \cos\left(x + \frac{\pi}{6}\right) ,$$

$$2\cos^2 x - \sin^2 x - \sin x - 6 = 0 , \sin\left(2x + \frac{\pi}{6}\right) = \cos\left(x + \frac{\pi}{3}\right)$$

$$4\cos^2 x + 2(1 - \sqrt{3})\cos x - \sqrt{3} = 0 , 3\cos x - 2 = 0$$

تمرين 12

حل في المجموعة $[0; 2\pi]$ المتراجحات التالية :

$$2\cos 2x - \sqrt{3} \geq 0 , \sqrt{2}\cos 3x + 1 \leq 0 , 2\cos x < 1$$

$$2\sin 5x + \sqrt{3} \geq 0 , \sqrt{2}\sin 4x - 1 \leq 0 , 2 < 1 , \cos 4x - \frac{1}{2} \geq 0$$

$$2\sin 4x - \sqrt{2} > 0$$

$$(\vec{OJ}, \vec{OB}) = (\vec{OJ}, \vec{OI'}) + (\vec{OI'}, \vec{OB})$$

$$= \frac{\pi}{2} - (\vec{OB}, \vec{OI'})$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$(\vec{OJ}, \vec{OB}) = \frac{\pi}{4}$$

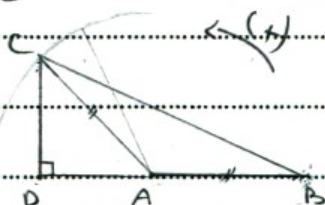
$$(\vec{OA}, \vec{OB}) = (\vec{OA}, \vec{OJ}) + (\vec{OJ}, \vec{OB})$$

$$= -(\vec{OJ}, \vec{OA}) + \frac{\pi}{4}$$

$$= \frac{\pi}{3} + \frac{\pi}{4}$$

$$(\vec{OA}, \vec{OB}) = \frac{7\pi}{12}$$

$$(\vec{AB}, \vec{AC}) = \frac{5\pi}{6}$$



(ب) حساب اقياس الزوايا

$$\bullet (\vec{BC}, \vec{BA}) = (\pi - \frac{5\pi}{6}) \times \frac{1}{2}$$

$$(\vec{BC}, \vec{BA}) = \frac{\pi}{12}$$

$$\bullet (\vec{CA}, \vec{CB}) = (\vec{BC}, \vec{BA}) = \frac{\pi}{12}$$

لان المثلث ABC متساوي الساقين

$$\bullet (\vec{AC}, \vec{CB}) = (-\vec{CA}, \vec{CB})$$

$$= (\vec{CA}, \vec{CB}) + \pi$$

$$= \frac{\pi}{12} + \frac{\pi}{12} = \frac{13\pi}{12}$$

$$3) \frac{35\pi}{2} = \frac{34\pi}{2} + \frac{\pi}{2} = 2(17)\frac{\pi}{2} + \frac{\pi}{2}$$

وهذا القيس الرئيسي α هو $\frac{\pi}{2}$

$$4) \frac{14\pi}{3} = \frac{12\pi}{3} + \frac{2\pi}{3} = 4\pi + \frac{2\pi}{3}$$

وهذا القيس الرئيسي α هو $\frac{2\pi}{3}$

الفتيل (سجل)

تمرين 4

$$(\vec{u}, \vec{ve}) = \frac{\pi}{3}$$

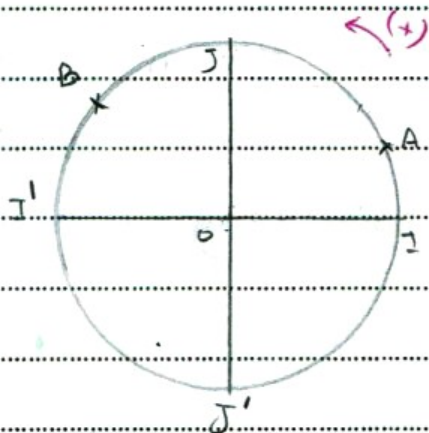
$$\bullet (\vec{v}, \vec{u}) = -(\vec{u}, \vec{ve}) = -\frac{\pi}{3}$$

$$\bullet (3\vec{ve}, 2\vec{u}) = (\vec{ve}, \vec{u}) = -\frac{\pi}{3}$$

$$\bullet (2\vec{ve}, -3\vec{u}) = (\vec{ve}, \vec{u}) + \pi = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$$

$$\bullet (-3\vec{v}, 7\vec{u}) = (\vec{ve}, \vec{u}) + \pi = \frac{2\pi}{3}$$

$$\bullet (-\vec{ve}, -\vec{u}) = (\vec{ve}, \vec{u}) = -\frac{\pi}{3}$$



تمرين 5

$$(\vec{OI}, \vec{OA}) = \frac{\pi}{6} ; (\vec{OI}, \vec{OB}) = \frac{3\pi}{4}$$

$$(\vec{OJ}, \vec{OA}) = -(\vec{OA}, \vec{OJ})$$

$$= -(\vec{OA}, \vec{OI}) - (\vec{OI}, \vec{OJ})$$

$$= (\vec{OI}, \vec{OA}) - (\vec{OI}, \vec{OJ})$$

$$= \frac{\pi}{6} - \frac{\pi}{2} = -\frac{2\pi}{6}$$

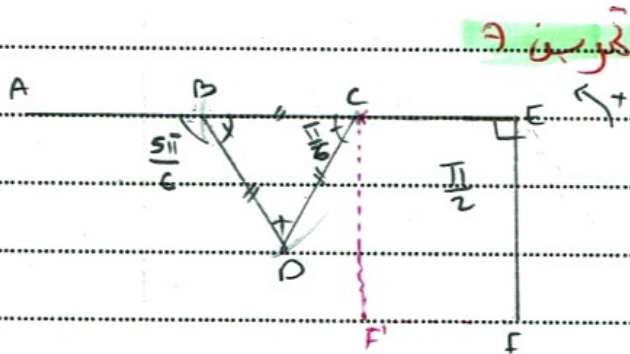
$$(\vec{OJ}, \vec{OA}) = -\frac{\pi}{3}$$

$$\sin(\vec{BD}, \vec{BA}) = \sin(\vec{BC}, \vec{BD})$$

$$= \frac{1}{2\sqrt{2+\sqrt{3}}} = \frac{\sqrt{2+\sqrt{3}}}{4+2\sqrt{3}}$$

$$\cos(\vec{BC}, \vec{BA}) = \cos(\vec{BC}, \vec{BD})$$

$$= \frac{-2+\sqrt{3}}{2\sqrt{2+\sqrt{3}}}$$



$$(\vec{BD}, \vec{EF}) = (\vec{BD}, \vec{BC}) + (\vec{BC}, \vec{EF})$$

$$= (\vec{BD}, \vec{BC}) + (\vec{BC}, \vec{CF})$$

$$= (\vec{BD}, \vec{BC}) + (-\vec{CB}, \vec{CF})$$

$$= (\vec{BD}, \vec{BC}) + (\vec{CB}, \vec{CF}) + \pi$$

$$= \frac{\pi}{6} + \frac{\pi}{2} + \pi$$

$$(\vec{BD}, \vec{EF}) = \frac{10\pi}{6} = \frac{5\pi}{3}$$

$$(\vec{CE}, \vec{EF}) = (-\vec{EC}, \vec{EF})$$

$$= (\vec{EC}, \vec{EF}) + \pi$$

$$= \frac{\pi}{2} + \pi = \frac{3\pi}{2}$$

$$(\vec{CE}, \vec{CD}) = -(\vec{CD}, \vec{CE}) = \pi - \frac{\pi}{6}$$

$$= -\pi + \frac{\pi}{6} = -\frac{5\pi}{6}$$

(3) حساب الأطوال

DC

لدينا المثلث ACD قائم في D و

$$(\vec{AC}, \vec{AD}) = \frac{\pi}{6}$$

و

$$(\vec{AC}, \vec{AD}) = \frac{DC}{AC}$$

وعليه

$$DC = \sin(\vec{AC}, \vec{AD}) \times AC$$

$$= \frac{1}{2} \times 2$$

أي

$$DC = 1$$

DA

$$\cos(\vec{AC}, \vec{AD}) = \frac{DA}{AC}$$

أي

$$DA = \cos(\vec{AC}, \vec{AD}) \times AC$$

$$= \frac{\sqrt{3}}{2} \times 2$$

$$DA = \sqrt{3}$$

أي

DB

$$DB = DA + AB = \sqrt{3} + 2$$

BC

حساب قوسين جيب

$$BC = \sqrt{DB^2 + DC^2}$$

$$= \sqrt{(2+\sqrt{3})^2 + 1^2}$$

$$= \sqrt{4+3+4\sqrt{3}+1}$$

$$BC = \sqrt{8+4\sqrt{3}}$$

$$BC = 2\sqrt{2+\sqrt{3}}$$

$$\sin \frac{\pi}{12}$$

$$= \frac{+\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = +\sqrt{3}$$

مربعین ۹

$$\begin{aligned} (\vec{AB}, \vec{BD}) &= (\vec{PA}, \vec{BD}) + \pi \\ &= \frac{5\pi}{6} + \pi = \frac{11\pi}{6} \end{aligned}$$

$$\cos x = 1/2$$

مربعین ۸

$$\bullet \sin x = \frac{1}{2}$$

یا

$$\cos^2 x + \sin^2 x = 1$$

$$\frac{1}{4} + \sin^2 x = 1$$

$$\sin^2 x = 1 - \frac{1}{4}$$

$$\sin^2 x = \frac{3}{4}$$

و

$$\sin x = -\frac{\sqrt{3}}{2} \text{ و } \sin x = +\frac{\sqrt{3}}{2}$$

$$\bullet \sin\left(\frac{\pi}{2} - x\right) = \cos x = \frac{1}{2}$$

$$\bullet \cos\left(\frac{\pi}{2} - x\right) = \sin x = +\frac{\sqrt{3}}{2}$$

$$\bullet \sin(\pi - x) = \sin x = +\frac{\sqrt{3}}{2}$$

$$\bullet \cos(\pi - x) = -\cos x = -\frac{1}{2}$$

$$\bullet \tan x = \frac{\sin x}{\cos x} = +\sqrt{3}$$

$$\begin{aligned} \tan\left(\frac{\pi}{2} - x\right) &= \frac{\sin\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right)} \\ &= \frac{1/2}{+\sqrt{3}/2} = +\frac{1}{\sqrt{3}} \end{aligned}$$

$$\tan(\pi - x) = \frac{\sin(\pi - x)}{\cos(\pi - x)}$$

$$\begin{aligned} \bullet A &= \cos\left(\frac{\pi}{2} + x\right) + \sin(\pi - x) \\ &+ \cos\left(\frac{\pi}{2} - x\right) - \sin(x) \\ &= -\sin x + \sin x + \sin x + \sin x \end{aligned}$$

$$A = 2 \sin x$$

$$\bullet B = -\cos x + \cos\left(\pi + \frac{\pi}{2} + x\right) + \sin(2\pi + \pi - x) + \sin\left(2\pi + \frac{\pi}{2} - x\right)$$

$$\begin{aligned} &= -\cos x - \cos\left(\frac{\pi}{2} + x\right) \\ &+ \sin(\pi - x) + \sin\left(\frac{\pi}{2} - x\right) \end{aligned}$$

$$\begin{aligned} &= -\cos x + \sin x + \sin x \\ &+ \cos x \end{aligned}$$

$$\bullet B = 2 \sin x$$

$$\bullet C = \cos\left(4\pi - \frac{\pi}{2} + x\right) + \cos\left(4\pi - \frac{\pi}{2} - x\right) + \sin x - \cos x$$

$$= \cos\left(-\frac{\pi}{2} + x\right) + \cos\left(-\frac{\pi}{2} - x\right)$$

$$= \sin x - \cos x$$

$$= \cos\left(-\left(\frac{\pi}{2} - x\right)\right) + \cos\left(-\left(\frac{\pi}{2} + x\right)\right)$$

$$= \sin x - \cos x$$

$$= \sin x - \sin x - \sin x - \cos x$$

$$C = -\sin x - \cos x$$

وهو المعادله تصبح

$$\cos(2x) = \cos\left(\frac{3\pi}{6}\right)$$

نكافي

$$\begin{cases} -2x = \frac{3\pi}{6} + 2k\pi \\ -2x = -\frac{3\pi}{6} + 2k\pi \end{cases}$$

$$\begin{cases} x = -\frac{3\pi}{12} - k\pi \\ x = \frac{3\pi}{12} - k\pi \end{cases}$$

$$\bullet \sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

نظم أ ب

$$\sin\left(x + \frac{\pi}{3}\right) = \sin\frac{\pi}{3}$$

نكافي

$$\begin{cases} x + \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi \\ x + \frac{\pi}{3} = \pi - \frac{\pi}{3} + 2k\pi \end{cases}$$

$$\begin{cases} x = 0 + 2k\pi \\ x = \frac{2\pi}{3} + 2k\pi \end{cases}$$

$$\bullet \cos\left(x - \frac{\pi}{4}\right) = \sin\left(3x + \frac{\pi}{2}\right)$$

نقوم بحل واحد الطرفين

$$\cos x = \sin\left(\frac{\pi}{2} - x\right)$$

نظم أ ب

$$\begin{aligned} \cos\left(x - \frac{\pi}{4}\right) &= \sin\left(\frac{\pi}{2} - x + \frac{\pi}{4}\right) \\ &= \sin\left(-x + \frac{3\pi}{4}\right) \end{aligned}$$

تمرين 10

حل المعادلات

$$\bullet \cos x = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

نظم أ ب
وهو المعادله تصبح

$$\cos x = \cos\frac{\pi}{6}$$

$$\begin{cases} x = \frac{\pi}{6} + 2k\pi \\ x = -\frac{\pi}{6} + 2k\pi \end{cases}$$

نكافي
أ ب

$$\bullet \cos x = -\frac{\sqrt{2}}{2}$$

$$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$$

نظم أ ب

$$\cos(\pi - x) = \cos(\pi - x) = -\cos x$$

$$\cos\left(\pi + \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

وهو المعادله تصبح

$$\cos x = \cos\frac{5\pi}{4}$$

$$\begin{cases} x = \frac{5\pi}{4} + 2k\pi \\ x = -\frac{5\pi}{4} + 2k\pi \end{cases}$$

نكافي
أ ب

$$\cos(2x) = -\frac{\sqrt{3}}{2}$$

$$\cos\left(\pi + \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

نظم أ ب

$$\sin^2 x - \sin x - 6 = 0$$

$$x = \sin x \quad \text{بفرض}$$

المعادلة تكون

$$x^2 - x - 6 = 0$$

$$\Delta = b^2 - 4ac$$

$$\Delta = 1 + 24$$

$$\Delta = 25$$

المعادلة تكون

$$x_2 = \frac{1-5}{2}$$

$$x_2 = -1$$

$$x_1 = \frac{1+5}{2}$$

$$x_1 = 3$$

$$\sin x = x$$

$$\sin x = 3 \quad \text{لأن } x = 3 \quad \text{لا}$$

و لا يوجد

$$\sin x = -1 \quad \text{لأن } x = -1 \quad \text{لا}$$

$$\sin \frac{\pi}{2} = -1 \quad \text{لأن}$$

المعادلة تكون

$$\sin x = \sin -\frac{\pi}{2}$$

بفرض

$$\begin{cases} x = -\frac{\pi}{2} + 2k\pi \\ x = \pi + \frac{\pi}{2} + 2k\pi \end{cases}$$

المعادلة تكون

$$S = \left\{ -\frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \right\}$$

و على المعادلة

$$\sin(-x + \frac{3\pi}{4}) = \sin(3x + \frac{\pi}{2})$$

بفرض

$$\begin{cases} -x + \frac{3\pi}{4} = 3x + \frac{\pi}{2} + 2k\pi \\ -x + \frac{3\pi}{4} = \pi - 3x - \frac{\pi}{2} + 2k\pi \end{cases}$$

$$\begin{cases} -4x = \frac{\pi}{2} - \frac{3\pi}{4} + 2k\pi \\ x = \pi - \frac{\pi}{2} - \frac{3\pi}{4} + 2k\pi \end{cases}$$

$$\begin{cases} x = \frac{\pi}{16} + \frac{1}{2} k\pi \\ x = -\frac{\pi}{4} + k\pi \end{cases}$$

$$\sin x + \cos x = 0$$

$$\sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = 0$$

$$\frac{\sqrt{2}}{2} \sin x + \frac{\sqrt{2}}{2} \cos x = 0$$

$$\sin \frac{\pi}{4} \sin x + \cos \frac{\pi}{4} \cos x = 0$$

$$\cos(\frac{\pi}{4} - x) = 0$$

بفرض

$$\cos(\frac{\pi}{4} - x) = \cos \frac{\pi}{2}$$

نحل المعادلة

تقریب 12 $[0, 2\pi[$

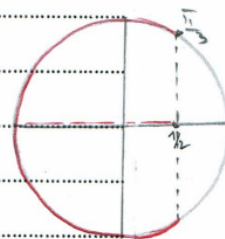
$$2 \cos x < 1$$

$$\cos x < \frac{1}{2}$$

ای

$$\cos x < \cos \frac{\pi}{3}$$

$x \in [\frac{\pi}{3}, \frac{5\pi}{3}]$



$$\sqrt{2} \cos 3x + 1 \leq 0$$

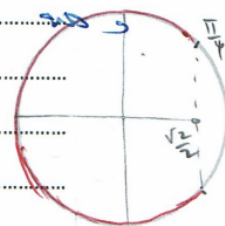
$$\cos 3x \leq \frac{1}{\sqrt{2}}$$

$$\cos 3x \leq \frac{\sqrt{2}}{2}$$

$$\cos 3x \leq \cos \frac{\pi}{4} \quad \text{و}$$

$$3x \in [\frac{\pi}{4}; \frac{7\pi}{4}]$$

$$x \in [\frac{\pi}{12}, \frac{7\pi}{12}]$$



$$2 \cos 2x - \sqrt{3} > 0$$

$$\cos 2x > \frac{\sqrt{3}}{2}$$

$$\cos 2x > \cos \frac{\pi}{6}$$

و

$$2x \in [0, \frac{\pi}{6}] \cup [\frac{11\pi}{6}, 2\pi]$$

و

$$x \in [0, \frac{\pi}{12}] \cup [\frac{11\pi}{12}, \pi]$$

